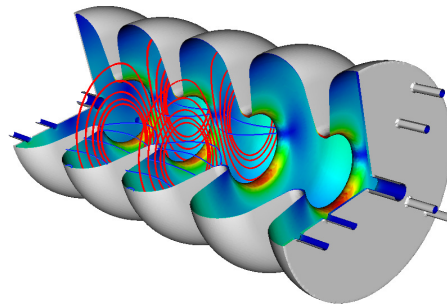


Applications of Frequency Extraction to Cavity Modeling

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Tech-X Corporation*, University of Colorado-Boulder[¶]

Collaborator: Leo Bellantoni, FNAL



*UCLA COMPASS Meeting
3 December 2008*



- **Background on Cavity Modeling**
- **Finite-Difference Time-Domain Simulations**
- **Frequency Extraction Algorithm**
 - **Filtered Excitation**
 - **Filter-Diagonalization**
- **Verification of Spherical Cavity**
- **Validation of A15 Cavity**
- **Conclusion**

Cavity Modeling



- **Verifying and validating EM codes is a crucial part of cavity modeling since it provides evidence of the effectiveness of the code**
- **COMPASS codes like Omega3p have made a concerted effort at V&V**
- **We focus in this talk on V&V efforts for Tech-X Corporation's VORPAL code**
- **VORPAL has been successful in the past at laser wakefield simulations and electron cooling**

Cavity Modeling

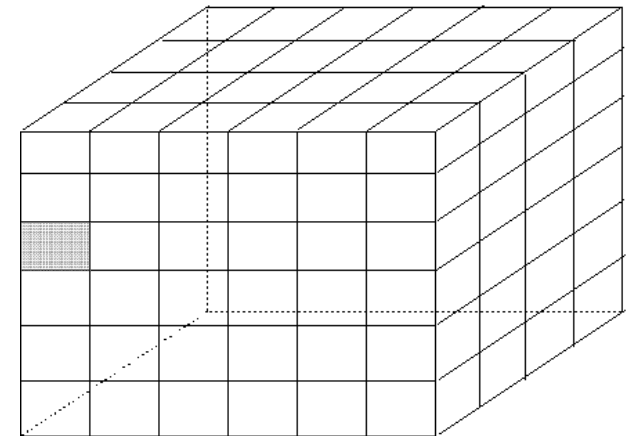
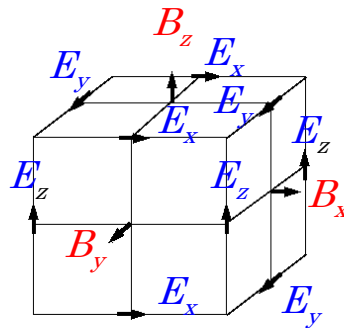


- **Machining is accurate to about 1 mil or 0.0254 mm**
- **Results in [Burt et al., 2007] showed frequencies to be sensitive to equatorial radius by about 80 MHz/mm for a deflecting cavity**
- **Machining can produce cavities with frequencies shifted by about ± 2 MHz from the original specs.**
- **Careful remeasurements after fabrication can be using simulations instead of bead pull experiments if the simulations are accurate.**

Finite-Difference Time-Domain



Maxwell's Equations



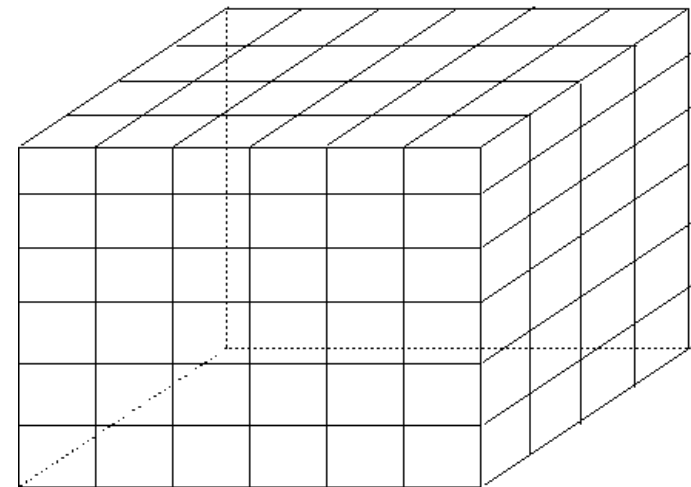
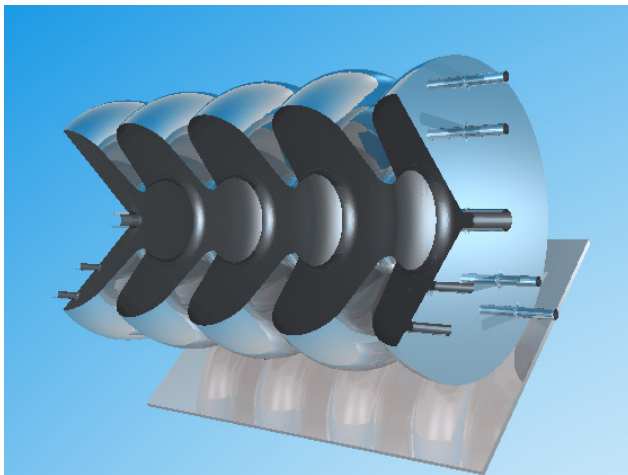
**“Rectangular
Grid”**

-
-
-

Finite-Difference Time-Domain



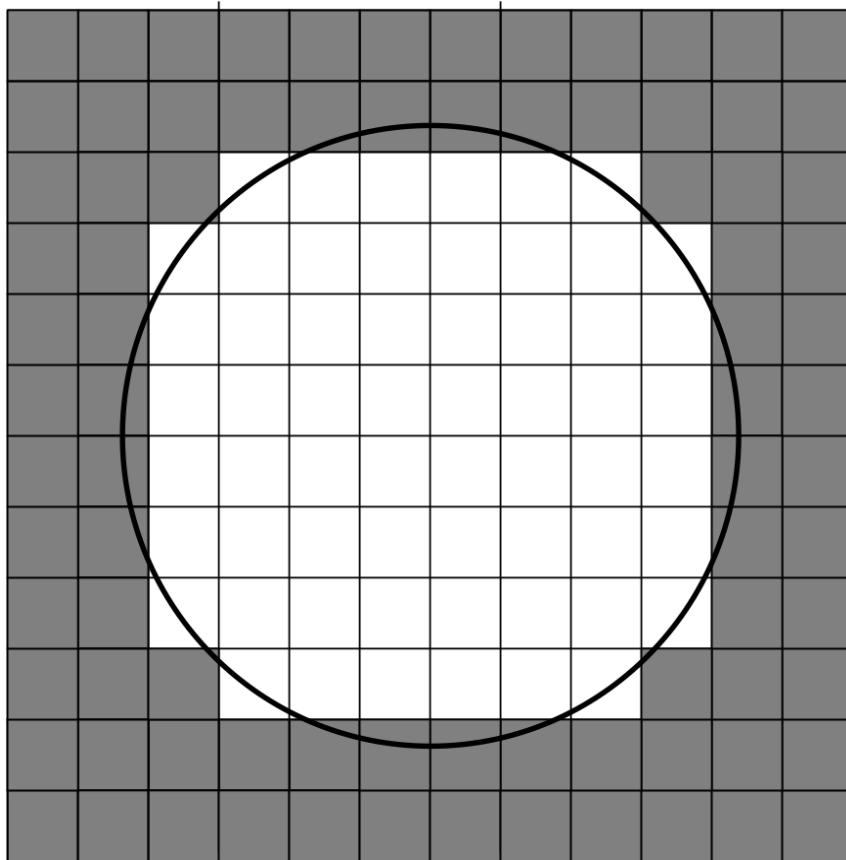
Embedded Boundary Methods



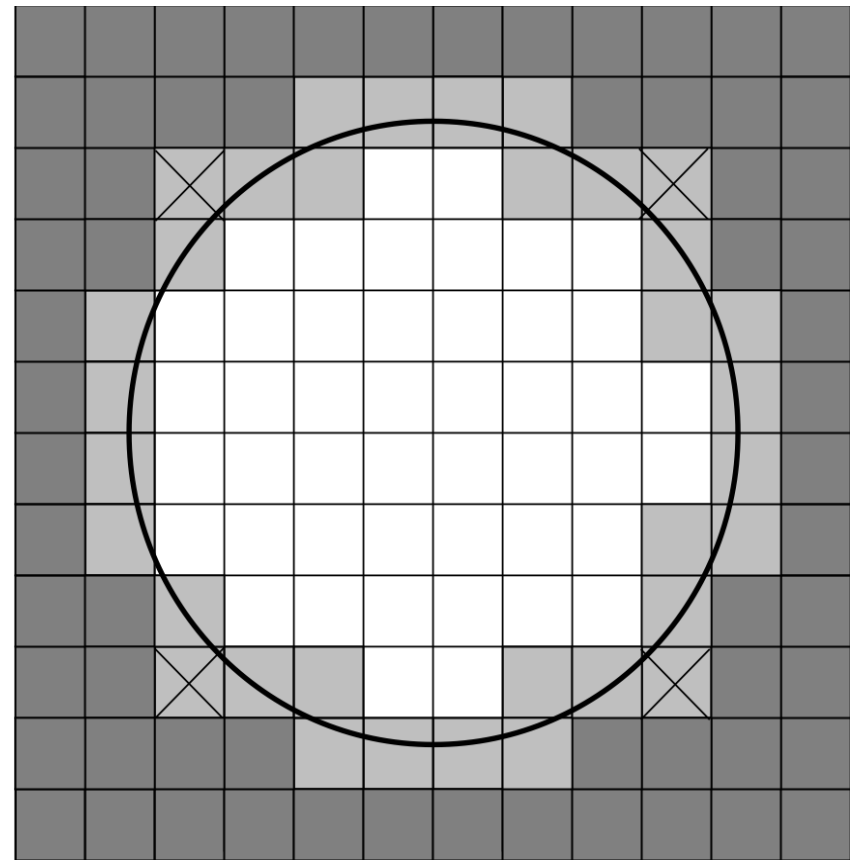
Embedded Boundary Methods

- Curved domains described analytically
- These domains are not represented by the logically rectangular domain in contrast to unstructured FE meshes
- There are three methods for representing contribution of curved boundaries for logically rectangular domains:
 - Stairstep
 - Dey-Mittra
 - Zagorodnov
- Stairstep and Dey-Mittra discussed on next page
- Zagorodnov only recently implemented

Finite-Difference Time-Domain



(a) Stairstep Approach



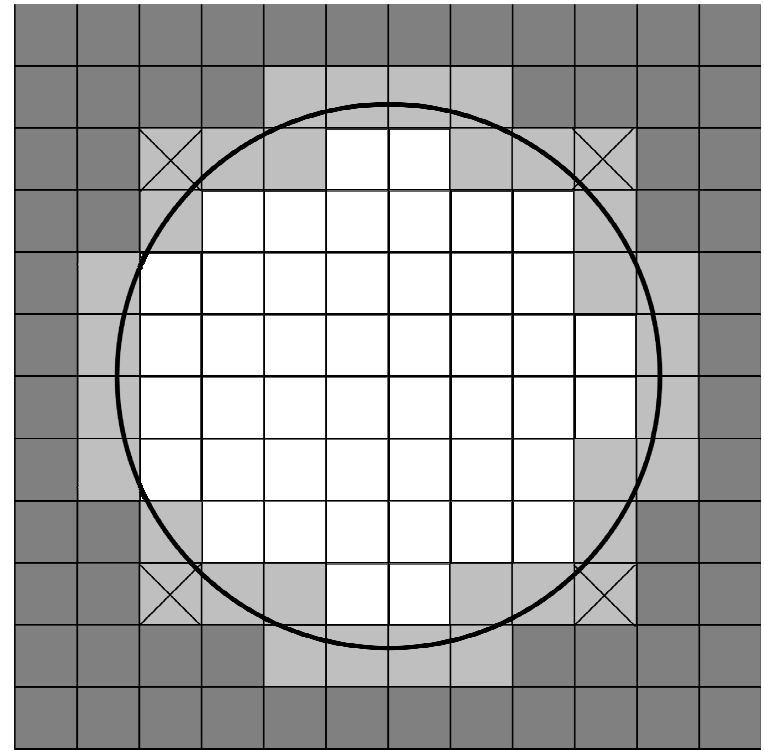
(b) Dey-Mittra Approach



- Only change Faraday update

Finite-Difference Time-Domain

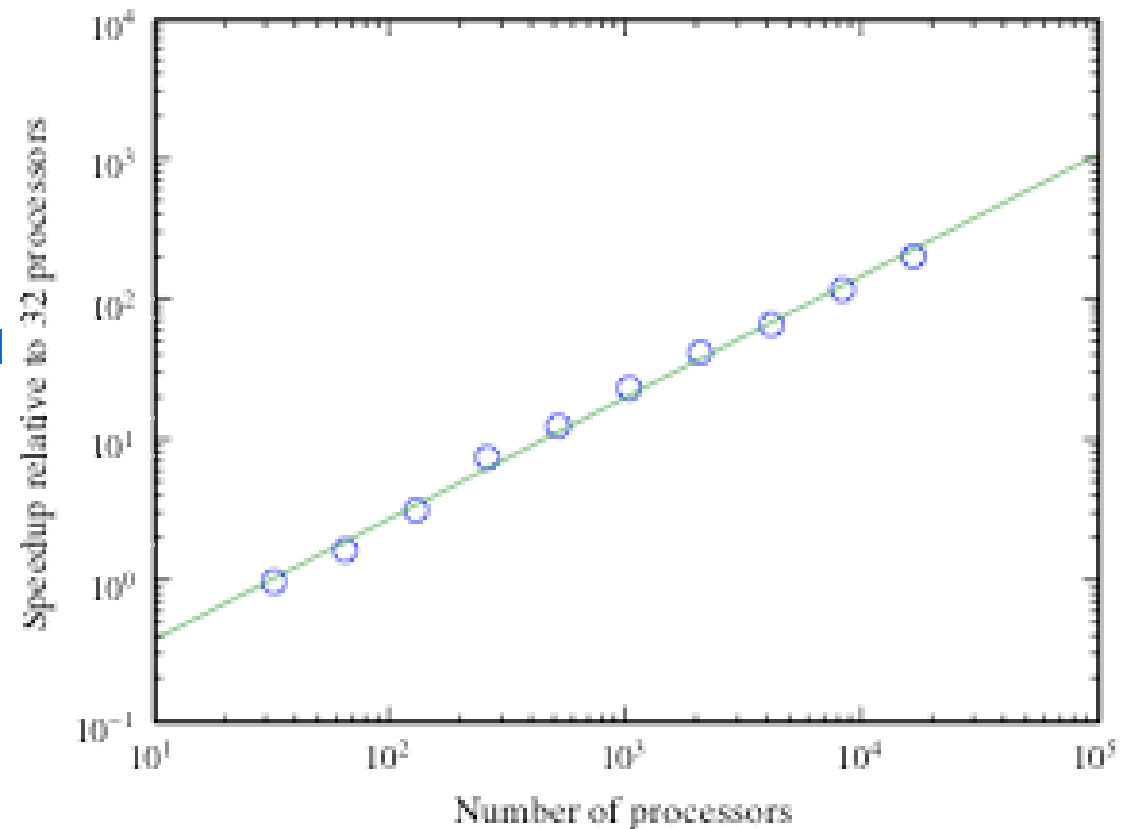
- FDTD is a second-order method
- Curved domains modeled using embedded boundary methods
- Embedded boundary method requires adjusting lengths (l_{ij} , l_{ik} , l_{jk}) and areas (a_{ij}) used in the **Faraday** update step
- Faces with small area excluded from computations to minimize the reduction in time-step due to CFL
- Method maintains second-order in time and space unless too many cells thrown out



VORPAL Computational Framework



- Based on the FDTD method
- Mainly uses the Dey-Mittra method for embedded boundaries
- Excellent scaling on >10000 processors of Franklin for EM problem with ~200 million grid points
- Load balancing and ADI methods currently being investigated for even better performance in the future



Frequency Extraction Algorithm



- Eigenvalue problems typically consist of constructing a large matrix system and using an iterative method to find the eigenvalues
- Robust eigenvalue solver is necessary to compute the eigenvalues in a reasonable time
- These methods require more memory (storing matrix and multiple vectors) and are generally less scalable than FDTD methods
- Goal is to construct an eigenvalue solver (or frequency extraction algorithm) that depends on FDTD methods which are very scalable and require minimal memory

Frequency Extraction Algorithm



- Use FDTD method as it scales well for massively parallel machines like the NERSC machine Franklin
- Extract frequencies through
 - Filter to desired modes
 - Determine subspace with SVD
 - Diagonalize in subspace
 - Get multiple modes at once

Extracting Degenerate Modes and Frequencies from Time Domain Simulations With Filter-Diagonalization [★]

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Abstract

A variant of the filter-diagonalization method, using targeted excitation to filter out unwanted modes, can extract exactly or nearly degenerate eigenmodes and frequencies from time-domain simulations. Excitation provides a particularly simple way to produce filtered states with already-existing time-domain simulations, while requiring minimal storage space. Moreover, using broader excitations that cover the entire range of desired frequencies requires just one-fifth as much computation as using narrow excitations. With this method, almost any time-domain code can be easily turned into an efficient eigenmode solver with little or no change to the code.

G.W Werner and J.R Cary, *J. Comp. Phys.*, **227**, 5200-5214, 10, (2008).

Frequency Extraction Algorithm



Consider

Then \hat{f}_k where

Use \hat{f}_k that vanishes for $t > T$, where T is the excitation time, i.e.,

where

For the range $\omega \in [\omega_0, \omega_0 + \Delta\omega]$, we use

Frequency Extraction Algorithm



Determine the approximate number of modes, M , in the range

Obtain L state vectors (\mathbf{s}_l) for $L > M$, the number of modes, which correspond to evaluation of the field at L times for $t > T$ and define $\mathbf{r}_l = \mathbf{H}\mathbf{s}_l$

Evaluate (\mathbf{s}_l) at P random points on the grid to obtain $P \times L$ matrix \mathbf{S} and the $P \times L$ matrix \mathbf{R} such that

\mathbf{R} and \mathbf{S} may be overdetermined so solve instead

Find the SVD of

Find the singular values of

Frequencies are calculated as

Frequency Extraction Algorithm



- Degeneracies (or near degeneracies) can be extracted with multiple simulations to generate the state vectors (\mathbf{s}_l)
- Once the state vectors are generated from FDTD simulations, the frequency extraction algorithm is quick ($< 1\text{min}$)
- Constructing the spatial mode patterns for each frequency also takes only several minutes depending on problem size
- Results in [Cary and Werner, 2008] verified method for 2D rectangular wave guide

Validation of Sphere



Simulation parameters



18 degree slice of a spherical cavity

Radius = 0.1 m

Grid size = 2 mm

Frequency range = 2 ~ 4 GHz

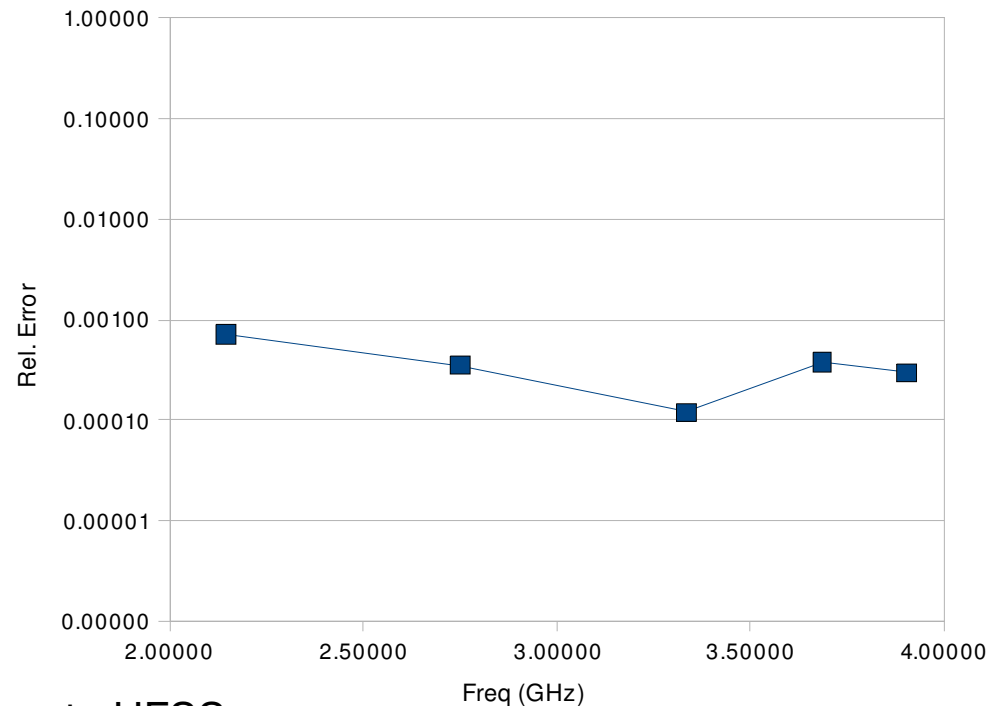
Expected modes (TE_{nmp})

TE ₁₀₁	2.14396 GHz
TE ₂₀₁	2.74995 GHz
TE ₃₀₁	3.33418 GHz
TE ₁₀₂	3.68598 GHz
TE ₄₀₁	3.90418 GHz

Validation of Sphere



Mode	Analytical (GHz)	Calculated (GHz)	Rel. Error
TE101	2.14396	2.14550	0.00072
TE201	2.74995	2.75091	0.00035
TE301	3.33418	3.33378	0.00012
TE102	3.68598	3.68458	0.00038
TE401	3.90418	3.90302	0.00030

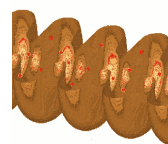
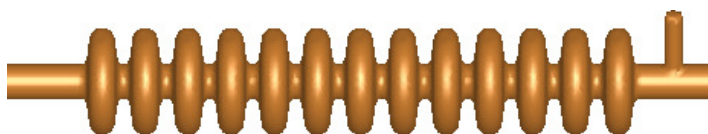


These preliminary results have similar accuracy to HFSS and Microwave Studio. Omega3p more accurate by a three orders of magnitude. (HFSS, Microwave Studio, and Omega3p results obtain from JLab. VORPAL results produced by Seah Zhou of Tech-X Corp.)

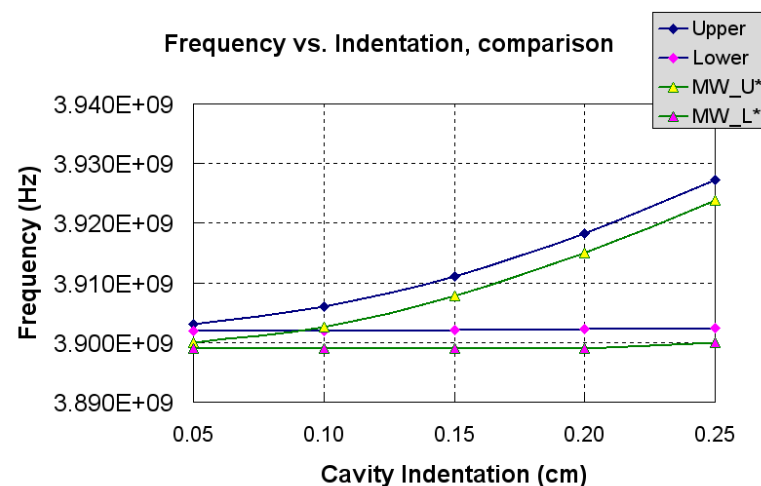
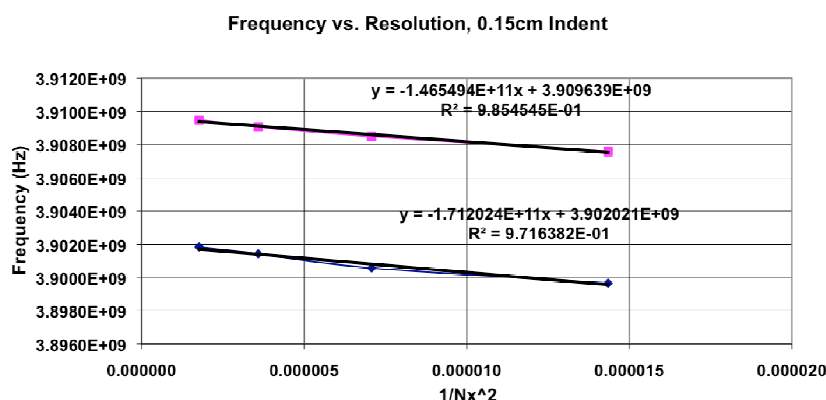
Validation of A15 Cavity

Background

- Compute frequencies for 9-cell crab cavity and compare to MAFIA/MWS



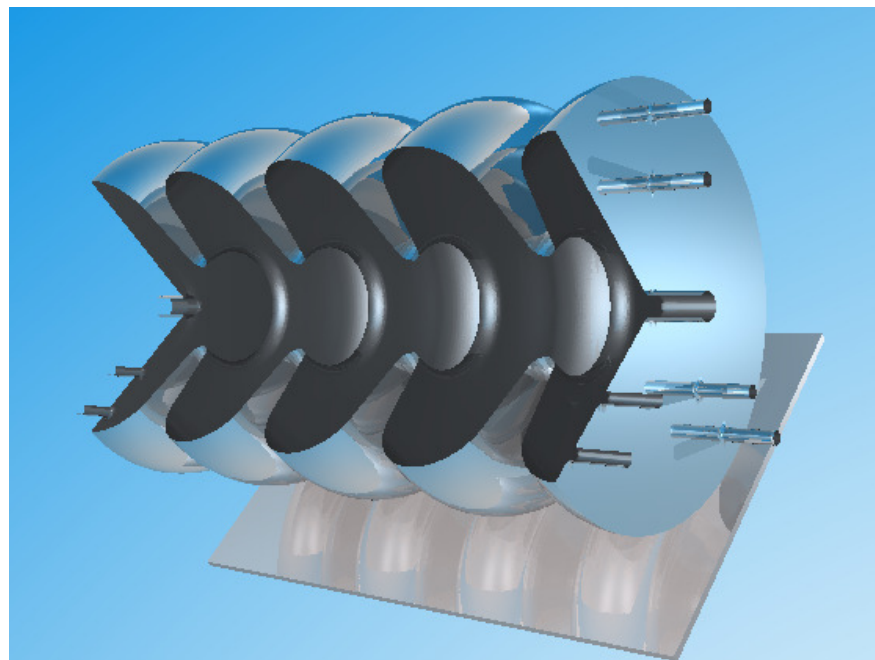
- Crab cavity squashed in the z-direction to eliminate degeneracies
- Simulations with up to 25 million cells
- Extrapolated results consistently differ from MAFIA/MWS by ~3 MHz



Validation of A15 Cavity



- A15 Cavity is an aluminum cavity fabricated at Fermilab in 1999
- Designed for development of a K⁺ beam
- It has been extensively tested, measured, and simulated
- Simulations performed by MAFIA considered computing frequencies of accelerating and deflecting modes
- Tech-X using VORPAL has concentrated on the deflecting (TM₁₁₀) modes from the A15

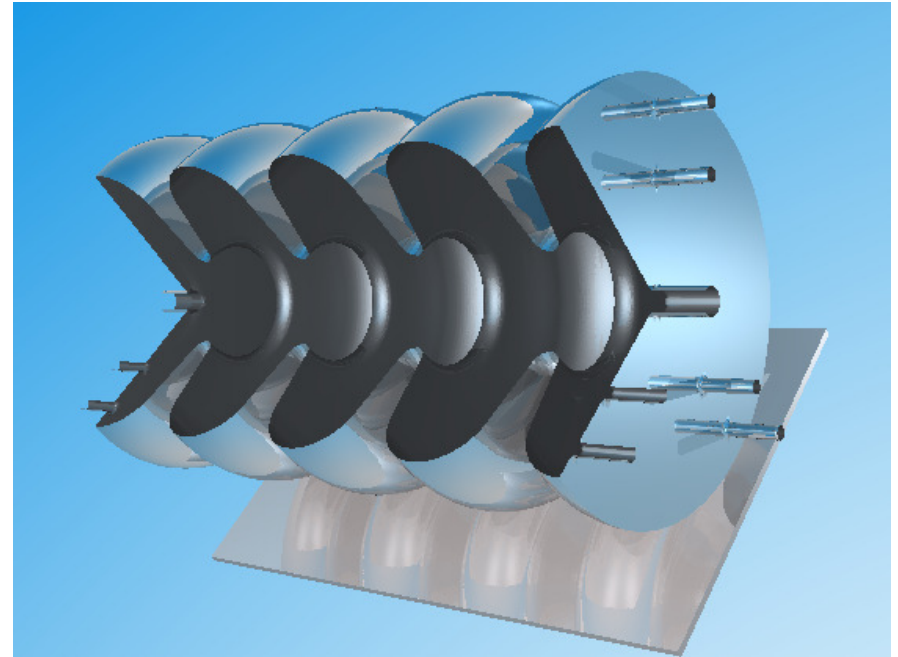


Validation of A15 Cavity



Equator Radius: 47.19 mm
Iris Radius: 15.00 mm
Cavity Length: 153.6 mm

Cavity contains end plate holes
used for bead pull experiments
and for creating dipoles



Five Deflecting Modes:

f_0	f_1	f_2	f_3	f_4
3902.810	3910.404	3939.336	4001.342	4106.164

Validation of A15 Cavity



Excitation Pattern:

Validation of A15 Cavity



Simulation Parameters:

Two simulations used to capture degeneracies

Excitation time: 100 periods @ 4 GHz

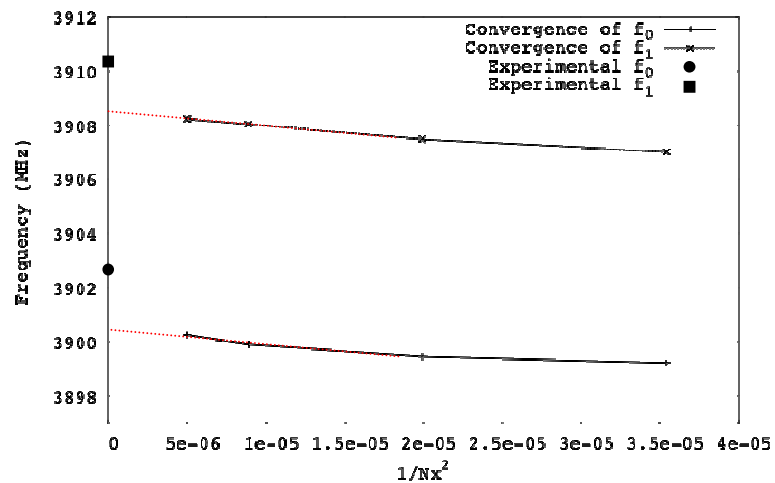
Total simulation time: 150 periods @ 4 GHz

Max number of grid points: ~20 million grid points

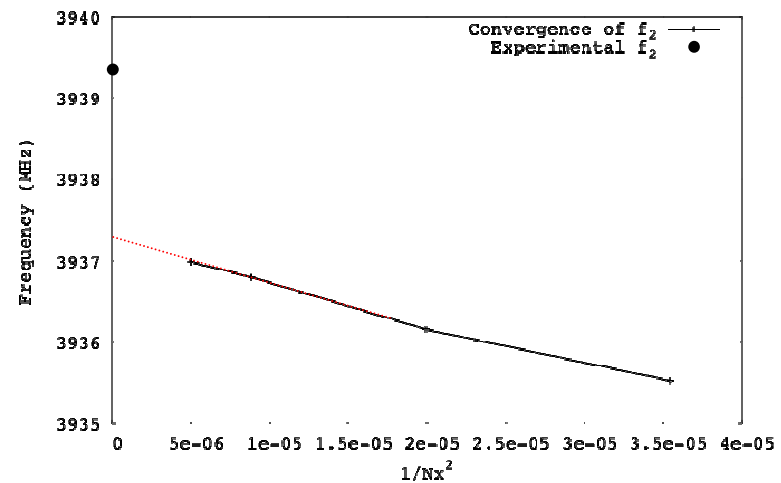
Max Total Time Steps: 437369 time steps

Validation of A15 Cavity

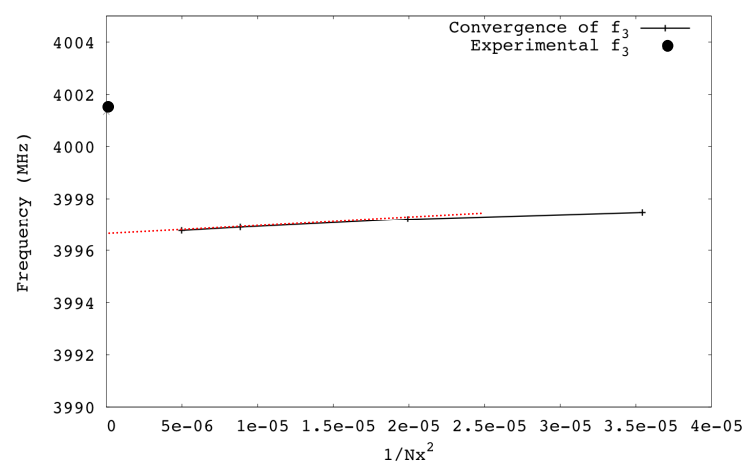
Convergence of 1st and 2nd Deflecting Modes of A15 cavity



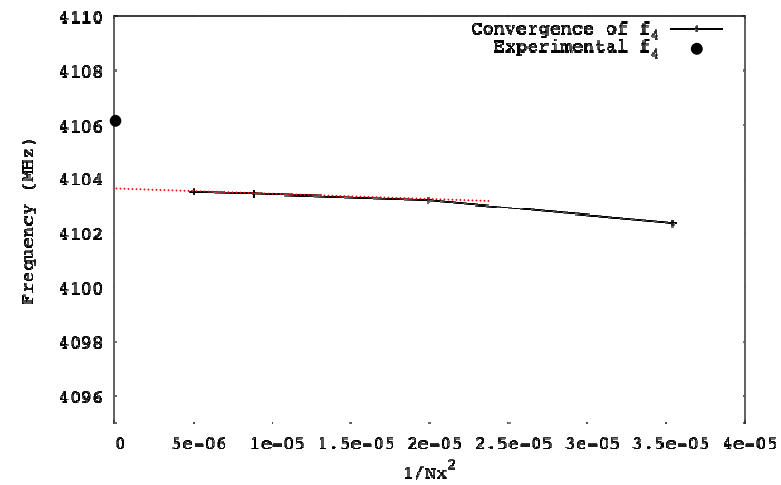
Convergence of 3rd Deflecting Modes of A15 cavity



Convergence of 4th Deflecting Modes of A15 cavity



Convergence of 5th Deflecting Modes of A15 cavity



Validation of A15 Cavity



Relative Error of Deflecting Modes Computed by MAFIA:

f_0	f_1	f_2	f_3	f_4
1.4e-3	1.3e-3	-----	-----	-----

Relative Error of Deflecting Modes Computed by VORPAL:

f_0	f_1	f_2	f_3	f_4
6.5e-4	4.9e-4	5.1e-4	1.2e-3	6.1e-4

Validation of A15 Cavity

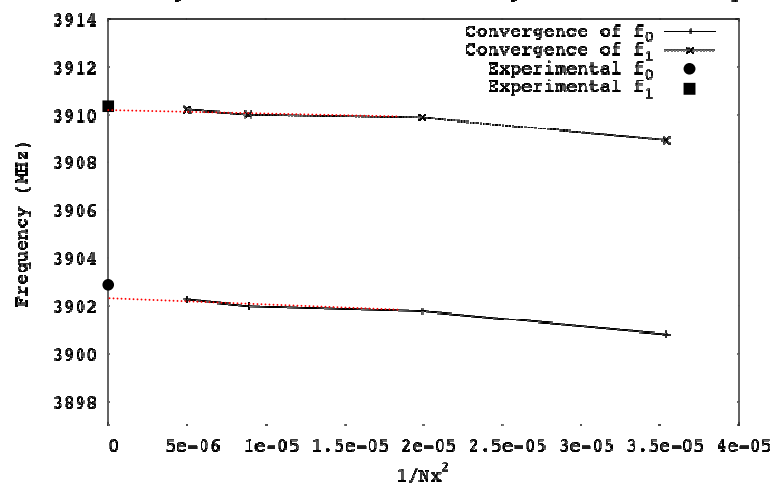


- **VORPAL was too low by 2 MHz for the p mode**
- **MAFIA was too low by 5 MHz for the p mode**
- **MAFIA calculations were too large on spacing between the p deflection mode and the next higher mode by 6.41% and VORPAL calculations were too large by 7.6%**
- **Possible causes for differences between calculations and experimental measurements:**
 - Failed to account for atmospheric conditions
 - End plate holes lead to frequency shift
 - Discrepancies between specs and machining

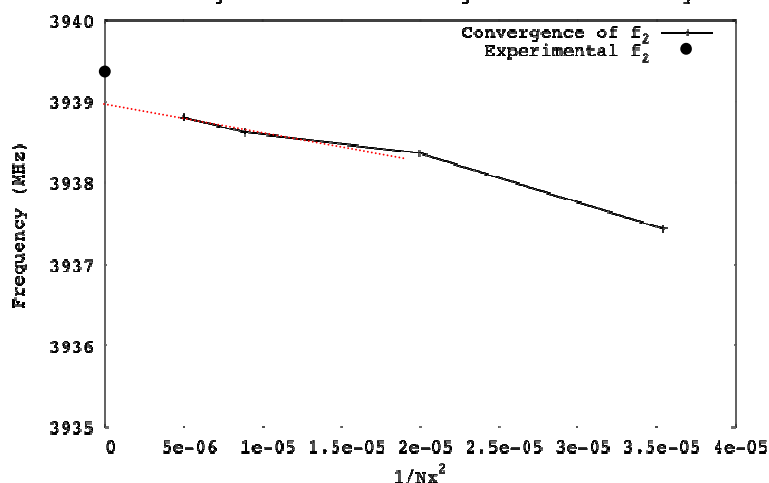
A15 Accelerator Cavity Computations



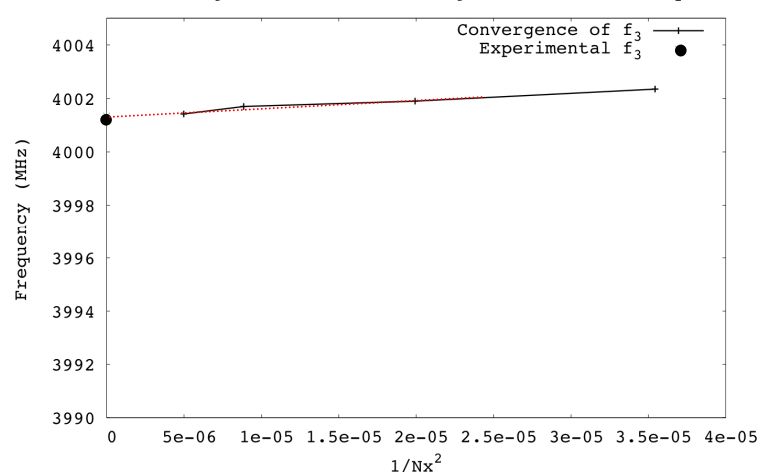
Convergence of 1st and 2nd Deflecting Modes of A15 cavity



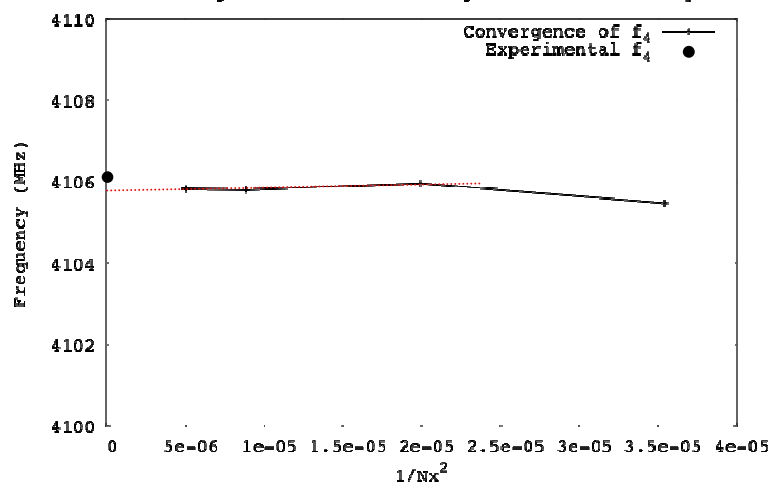
Convergence of 3rd Deflecting Modes of A15 cavity



Convergence of 4th Deflecting Modes of A15 cavity



Convergence of 5th Deflecting Modes of A15 cavity



Validation of A15 Cavity



Relative Error of Deflecting Modes Computed by VORPAL for original equatorial radius:

f_0	f_1	f_2	f_3	f_4
6.5e-4	4.9e-4	5.1e-4	1.2e-3	6.1e-4

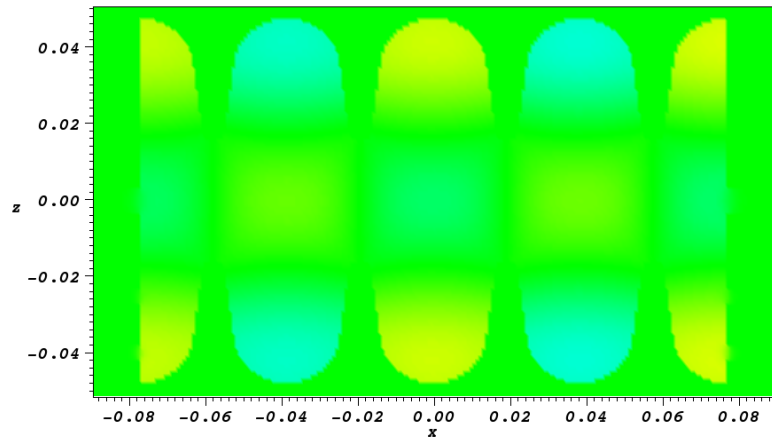
Relative Error of Deflecting Modes Computed by VORPAL for 0.03 mm smaller equatorial radius:

f_0	f_1	f_2	f_3	f_4
5.6e-5	1.4e-5	7.3e-5	7.0e-5	7.6e-5

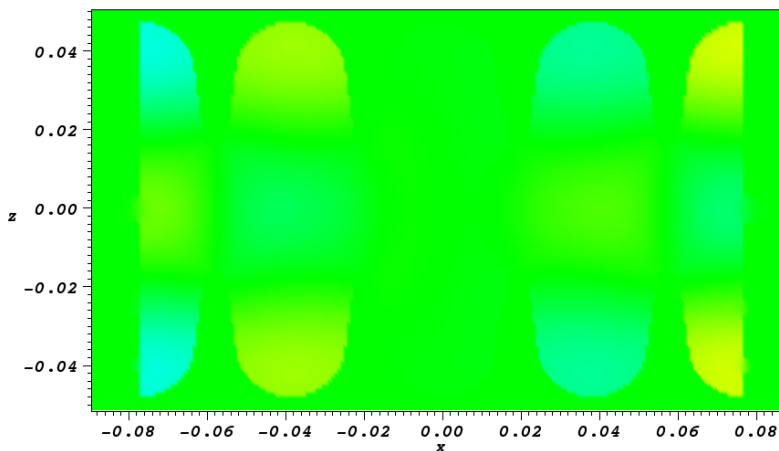
A15 Cavity Computations



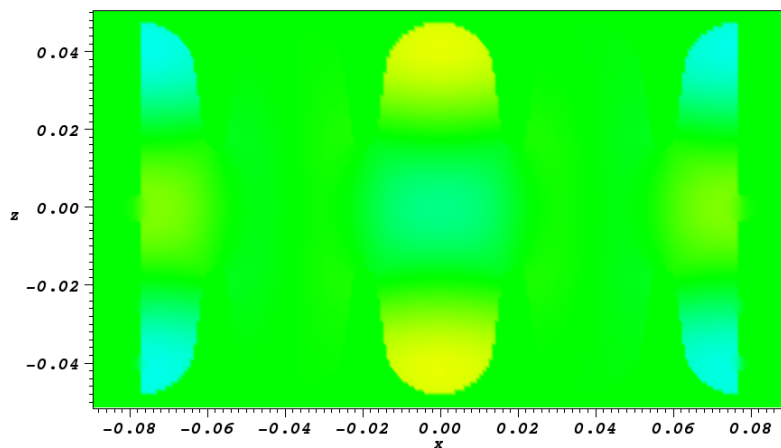
3902.810 MHz (p mode)



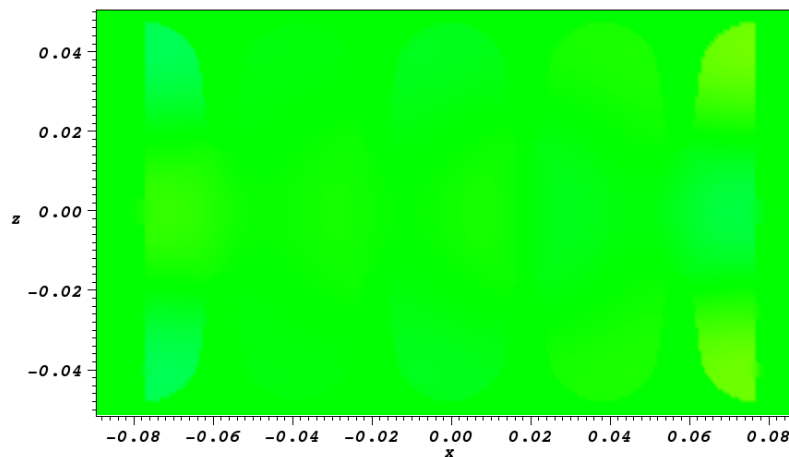
3910.404 MHz



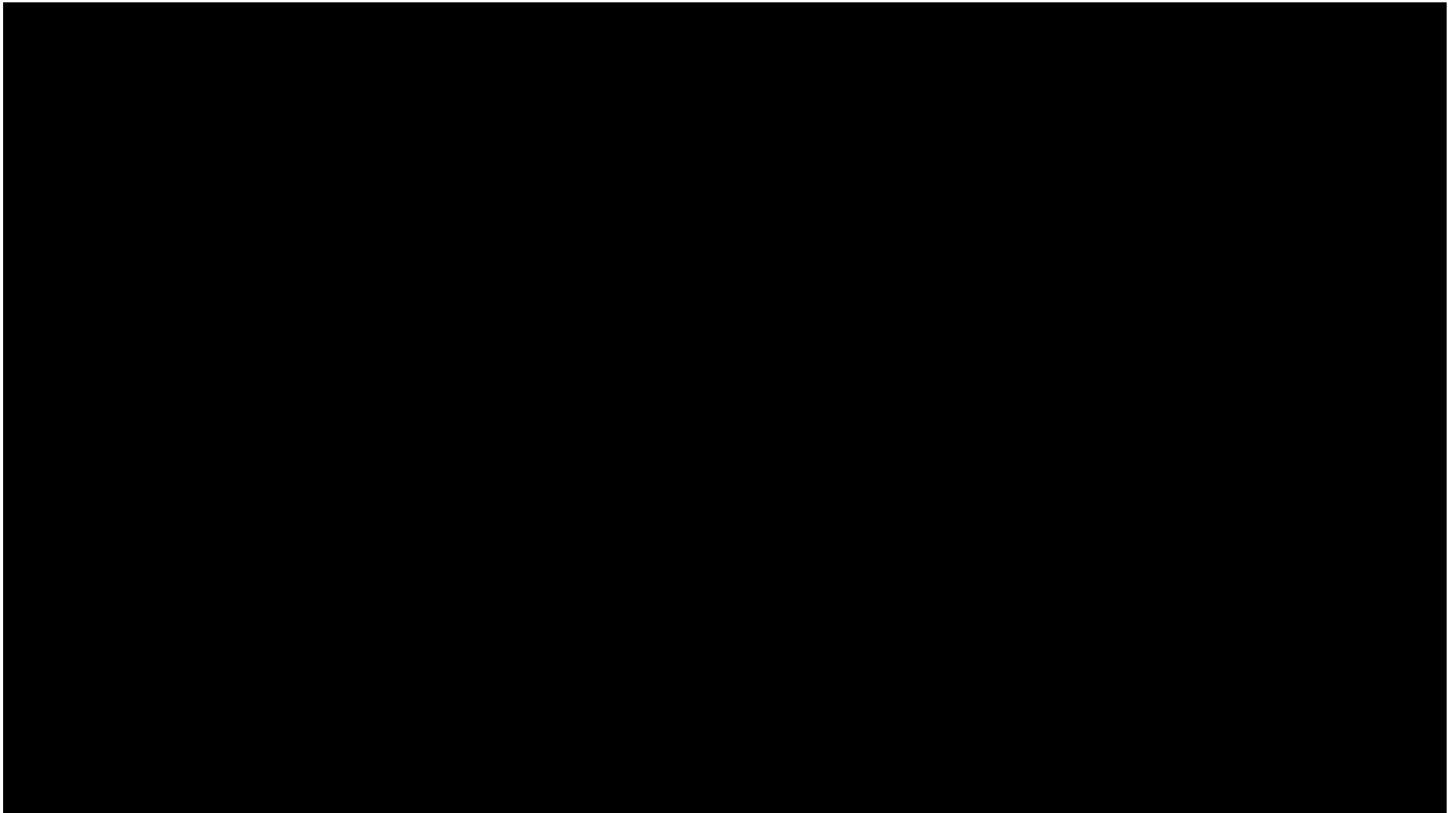
3939.336 MHz



4001.342 MHz



Complete Picture of p deflection mode



Final Remarks



- **Thanks to Leo Bellantoni at FNAL for assisting on verification study**
- **Working with Jlab on further validation for sphere and examining maximum value of B field on surface**
- **We are currently working on a paper which will be submitted soon showcasing this work**
- **Future topics consist of using algorithm in an optimization loop for cavity design.**